Lesson 1: Experimental and Theoretical Probability

Probability is the study of randomness. For instance, weather is random. In probability, the goal is to determine the chances of certain events happening. For instance, a meteorologist may determine that there is a 50% chance of rain tomorrow. We all have an intuitive sense that certain events are likely (a sunny day in July is likely) or unlikely (winning the lottery is unlikely), but the study of probability uses mathematics to assign numbers to these events. Using rules of probability, we could say there is a 98% chance of a sunny day in July, or a 0.1% chance of winning the lottery.

Why study probability in a statistics class? The main reason is that inferential statistics depends on random samples. For example, if a random sample of 100 people are chosen, what is the probability that the sample mean will be within 2 standard deviations of the population mean? After studying the basic rules of probability, we will be able to answer questions such as these.

We will start this chapter on probability by investigating simple examples, such as rolling dice and flipping coins.

Dice Rolling

Roll 2 dice.

Then add the number of dots facing up. For example, if you see:

Then the result is 8.

**Question 1:** Suppose you’re playing a game, and you win if you roll an 8. You want to know the probability of getting an 8. What are all the ways you can get an 8?
**Question 2:** Now, how many possible results are there when you roll 2 dice? Ok, I’ll make it easy for you. Look at the diagram below:

![Diagram showing all possible combinations of rolling 2 dice.](image)

This diagram shows every possible combination when rolling 2 dice. How many combinations are there?

**Question 3:** To compute a probability, count how many ways you can get an 8, and divide by all the possible outcomes:

**Question 4:** Now, let’s test this theory. Roll the dice 50 times, and keep track of how many times you roll an 8

**Question 5:** Let’s combine the entire class’s results together and see what percentage of the rolls were actually 8. How close is the percentage to your answer in question 3?

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**YOU NEED TO KNOW**

There are three ways to give the answer to a probability question:

a) In fraction form. For example: “the probability of rolling an 8 is 5/36”

b) In decimal form. For example: “the probability of rolling an 8 is about 0.1389”

c) In percent form: For example: “the probability of rolling an 8 is about 13.89%”

**Question 6:** Compute the probability of rolling a ten. Use the dice chart as a guide. Count the ways to get a ten, then divide by 36. Write the answer in all three forms (fraction, decimal, percent)

**Question 7:** Now, time for some vocabulary. Write down a guess for each italicized word.

  a) Rolling the dice is a **random trial** (also known as a **random experiment**, or **chance experiment**)

  b) Getting an 8 is an **event**

  c) An **outcome** is a 5 on the black die and 3 on the white die.

  d) All 36 combinations of dice rolls is a **sample space**
e) The **theoretical probability** of getting an 8 is 5/36, or 0.1389

f) The **empirical probability** of getting an 8 is (insert your answer from question 5)

g) The result from the dice is a **random variable**

h) A **success** is rolling an 8

i) A **failure** is not rolling an 8

**Time for some notation:**

A capital P is used to represent probability, and the event is put in parentheses. For example, if you want to express the idea, “the probability of rain is 0.7”, you could write:

\[ P(\text{Rain}) = 0.7 \]

You could also write \( P(R) = 0.7 \), if it’s understood that R represents “event that it rains tomorrow”. You could also write \( P(R) = 70\% \), or \( P(R) = 7/10 \)

If you’re dealing with random variables, the event could be written using mathematical symbols. For example:

\[ P(X = 8) \] could mean the probability of rolling an 8 on 2 dice.

\[ P(x > 8) \] could mean the probability of getting a result that is more than 8
Lesson 2  
Calculating Theoretical and Empirical Probabilities, and the Law of Large Numbers.

In the last lesson, you computed the probability of getting certain results on dice. Now it’s time for a formula that is useful in more general situations.

**Rule for calculating theoretical probability**
When all outcomes are equally likely, then the theoretical probability of an event can be calculated by the following formula:

\[
P(A) = \frac{\text{number of outcomes in } A}{\text{number of all possible outcomes}}
\]

**Question 8**: Suppose you have a bag containing 3 red marbles, 4 white marbles, and 2 blue marbles. The marbles all have the same size and weight.

a) How many outcomes are there? (hint: the answer is *not* 3)

b) In your opinion, is each outcome equally likely?

c) If you pick one marble at random, what is the probability of getting a white marble? Use the formula to answer the question

d) If you pick a marble at random, what is the probability of getting a red marble?

**Coin Flipping**
If you flip a coin, there are two possible events: the coin shows heads or the coin shows tails  
Let \( H = \) heads  
Let \( T = \) tails

**Question 9**: Determine \( P(H) \) and explain what it means

**Question 10**: Is your answer theoretical probability or experimental (empirical) probability?

**Question 11**: If you flip a coin 20 times, approximately how many times would you expect to see heads?

Theoretical probability gives you an estimate of how often an event will occur. The actual number of times an event occurs usually will be a little higher or a little lower than what the theoretical probability predicts.
Empirical probability is useful for a number of reasons. First, it’s a useful way to verify a theoretical probability. It is also useful in situations where it is difficult or impossible to calculate the theoretical probability (for example, when the outcomes are not equally likely, or if there are too many outcomes to count).

**Example:** In the past year, a salesperson has contacted 450 potential clients. Out of all these contacts, 68 of these resulted in a sale. What is this salesperson’s probability of success (i.e. probability of making a sale)?

**Solution:** divide the number of successes by the total number of attempts: 68/450 = 0.15111

**Answer:** The probability of success is about 15%

**Question 12:** Now, with the class, we will compute an empirical probability. Flip a coin 20 times, and then record how many heads and how many tails you get:
- number of heads (out of 20):
- number of tails (out of 20):

**Question 13:** Now, we’ll combine the results from the whole class. Compute P(H) and P(T)

This exercise brings up a very important rule of probability: the **Law of Large Numbers**

**Law of Large Numbers**
As the number of trials increases, the empirical probability gets closer to the theoretical probability.

For example, if you flip a coin 20 times, it’s difficult to predict how many heads you will get. However, if you flip a coin 2000 times, you can fairly confidently predict roughly 1000 heads and 1000 tails.

The law of large numbers explains how gambling casinos can reliably predict how often a player will win a game. For instance, if a game has a 10% chance of winning, and only 5 people play the game, the casino operators cannot predict how many people will win. Maybe none, maybe 1 or maybe all 5. However, if 5000 people play the game, then almost certainly about 50 people will win. (10% of 5000 = 50 people)
**Question 14:** If you roll two dice, the probability of getting a seven is about 17%. In other words, \( P(X=7) = 0.17 \). If you were to roll dice 5000 times, approximately how many times would you expect to roll a seven?
Lesson 3: More practice on calculating theoretical probabilities

In general, to calculate theoretical probabilities, there are three steps:

1. Make the sample space (list out all the outcomes). Make sure each outcome is equally likely
2. Count how many of those outcomes will create the desired event
3. Divide

Flipping more than one coin:

Suppose you flip a coin three times. There are 8 possible outcomes. The sample space consists of the eight possible outcomes listed below:

HHH  HHT  HTH  HTT  THH  THT  TTH  TTT

Here are some events associated with this experiment.

Example: Calculate the probability of the event: “Exactly 2 heads come up”.

Solution: Look at the sample space, and count all the ways the event can occur.

So, \( P(\text{exactly 2 heads come up}) = \frac{3}{8} \)

You could also write: \( P(X=2) = \frac{3}{8} \)

Question 15: Calculate probability of the event: “Exactly 3 tails come up”. Use the diagram below.

Question 16: Calculate the probability of the event: “A tail comes up on the first flip”. Use the diagram below:
Definition
If all of the possible outcomes of a chance experiment have the same probability of occurring we refer to this situation as “equally likely outcomes”.

Question 17: If you flip a coin three times, each of the 8 possible outcomes is equally likely. Why?

Useful tip #1:
When creating the sample space, it can get confusing to count every possible combination of outcomes. One way to count all the outcomes in an organized way is to make a tree diagram. For example, when counting all the outcomes of tossing 3 coins, the tree diagram would look like this:

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Useful Tip #2
When a random trial is being repeated, multiply the number of outcomes together to find the total number of combinations.

Example: Each time a coin is flipped, there are two outcomes. If two coins are flipped, there are $2 \times 2 = 4$ outcomes. If three coins are flipped, there are $2 \times 2 \times 2 = 8$ outcomes. Each time another coin is added, the number of outcomes doubles.

Example: In a lottery game, three digits are picked at random (the same number can be repeated). For instance, 4-2-2 is one possible combination. How many combinations are there, and what is the probability of getting the combination 0-0-0?

Solution: There are 3 digits, so imagine filling in the blanks with 3 random digits:
There are 10 choices for the first digit (0-9), 10 choices for the second digit, and 10 choices for the third digit. So altogether, there are $10 \times 10 \times 10 = 1000$ combinations.

The probability of getting 0-0-0 is 1 out of 1000, or $\frac{1}{1000} = 0.001$

**Answer:** $P(0-0-0) = 0.001$

**Question 18:** A couple plans to have three children. What is the probability that all three children will be girls?

**Question 19:** A license plate has 3 letters and 4 digits. For example, ABC 1234. How many combinations of license plates are there? (it’s a lot!). What is the probability of randomly getting the license plate AAA 1111?
Lesson 4: Probability Rules

There are some basic rules that are very helpful in calculating various probabilities:

### PROBABILITY RULES

There are some basic rules that are very helpful in calculating various probabilities:

Let A and B be events

1. Any probability must be between 0 and 1 (or 0% and 100% in percentage form)

   For example P(A) = 0.9 or P(A) = 56/82 are valid, but P(A) = 1.2 or P(A) = -0.2 are NOT valid.

   If P(A) = 0, then event A is impossible
   If P(A) = 1, then event A is guaranteed to happen

2. if A and B are mutually exclusive (cannot happen at the same time), then P (A or B) = P(A) + P(B)

   For example, suppose when the phone rings, there is a 10% chance that it’s your mother calling, and a 15% chance that it’s your father calling, then there is a 25% chance that it’s your mother or father calling. (assuming they’re not calling you at the same time)

3. If a list of events are all mutually exclusive, and then the sum of all probabilities is 1

   For example, if events A, B, C and D is the complete set of possible events, and they cannot happen at the same time, then P(A) + P(B) + P(C) + P(D) = 1

4. P (not A) = 1 – P(A) (or 100% – P(A) in percentage form)

   In other words, to know the probability of event A not happening, subtract from 1. For example if the probability of rain is 0.6, then the probability that is doesn’t rain is 0.4

In probability, “event A not happening” is usually referred to as the complement of A. There are several symbols used for complement. Here are a few:

A’
A^c
~A
not A

**Example**: Write a statement which is the complement of “a student earns a grade of A on their exam”.

**Answer**: “A student does not earn a grade of A on the exam”
Example: If there is a 40% chance that a student earns an A, what is the probability that he/she does not earn an A?

Solution: Let $A = \text{the student earns an A on the exam}$, then $P(A) = 0.4$, and $P(\text{not } A) = 1 - 0.4 = 0.6$

Answer: There is a 60% chance that the student does not earn an A on the exam.

Question 20: Suppose you have a bag containing 3 red marbles and 2 blue marbles and 5 white marbles. Find the probabilities below:

a. $P(\text{red}) =$

b. $P(\text{not red}) =$

c. $P(\text{red}) + P(\text{not red}) =$

d. $P(\text{red or white})$

e. $P(\text{not blue})$

f. $P(\text{blue or red})$
Lesson 5: Independence and Dependence

This lesson introduces another rule for probability, which involves combining events together. But the rule depends on whether the events are *independent* or *dependent*.

Two events are called *independent* if one does not affect the other. For example, dice rolls and coin flips are independent. If a coin is flipped and the result is heads, on the next flip the probability of heads is still 50%. In fact, if a coin is flipped 7 times in a row and all 7 flips are heads, the probability of heads on the 8th flip is still 50%. If events are independent, the probability does not change.

Some gambling games, such as slot machines and lotteries, are independent. However, there is a common misconception among gamblers related to independence. After losing many games in a row, the gambler believes he or she is “due” for a win. This is called the *gambler’s fallacy*. But in reality, the probability of a win does not change. For example, if the probability of a win is 10%, and the gambler has lost a game 9 times in a row, then the probability of a win on the 10th game is still 10%.

When events are independent, there is a useful rule for determining the probability of a combination of events occurring.

**YOU NEED TO KNOW**

If events A and B are *independent*, then the probability that both of these events occur is the product of the individual probabilities.

\[ P(A \text{ and } B) = P(A) \cdot P(B) \]

This rule can be extended to three or more independent events.

*Example*: What is the probability of rolling a fair die two times and getting two sixes?

*Solution*: Since each roll is independent, the probabilities remain the same.

\[ P(2 \text{ sixes}) = \frac{1}{6} \times \frac{1}{6} = \left( \frac{1}{6} \right)^2 = \frac{1}{36} \text{ or } 0.0278 \]

*Answer*: the probability of getting two sixes is about 2.78%

Notice that we could have done this the “long way”, by listing out every combination. But this multiplication rule serves as a useful short-cut.

*Example*: What is the probability that an adult American male is at least 6 feet tall and has an intelligence quotient (IQ) of at least 120, given that the probability a man is at least 6 feet tall is 0.145 and the probability of an adult having an IQ of at least 120 is 0.089?
Solution: Since height and IQ are independent events, you can simply multiply the two individual probabilities:

\[
\begin{align*}
P(\text{at least 6 feet tall}) &= 0.145 \\
P(\text{IQ at least 120}) &= 0.089 \\
P(\text{at least 6 feet tall and IQ at least 120}) &= 0.145 \times 0.089 = 0.0129
\end{align*}
\]

Answer: The probability that an adult American male is at least 6 feet tall and having an IQ at least 120 is about 0.0129.

Question 21: Rolling a dice and tossing a coin are independent events. If you were to put a dice and a penny in a cup and spill them out on a table, what is the theoretical probability of getting the number 2 on the dice and tails on a penny? Write you answer in the same format as the example.

Question 22: If we have two dice and we want to know the probability of getting the number 2 on both, can we use the method described here to find the theoretical probability? Why/why not?

Question 23: If we have one dice and we want to know the probability of getting an even number on two consecutive rolls, can we use method described here to find the theoretical probability? Why/why not?

Dependence

Not all events are independent. If the probability of one event is influenced by another event, it is called dependent. For example, consider drawing marbles out of a bag.

Question 24: Suppose a bag contains 4 red marbles and 6 green marbles. What is the probability of getting a green marble?

Question 25: Suppose that a green marble actually was picked, and not put back in the bag. Now, what is the probability that the next marble will be green?

As you can see, in this case, the probability of getting a green marble changed. The outcome of the first trial affected the probabilities of the next trial. The probability of getting a green marble on the second pick is dependent on the result of the first pick. Note, however, that if the green marble had been put back before the second pick, then the events would have been independent.

If events are not independent, you cannot simply multiply the individual probabilities. You must consider the additional information given and how that affects the probability of the additional events.

Example: What is the probability a three-person subcommittee contains all females if the members are randomly selected from a group with five males and five females?

Solution: Since the probability of choosing a female changes with each additional selection, the events are not independent. For the selection of the first person, there are 10 people in the group: five females and five males, so the probability of selecting a female is
For the selection of the second person, there are nine people left in the group: four females and five males, so the probability of selecting a female is

\[ P(\text{female 2nd selection}) = \frac{4}{9} \]

For the selection of the third person, there are eight people left in the group: three females and five males, so the probability of selecting a female is

\[ P(\text{female 3rd selection}) = \frac{3}{8} \]

So, the probability of selecting an all-female committee is equal to the product of the individual probabilities above:

\[ P(\text{three females selected}) = \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{60}{720} = \frac{1}{12} = 0.08333 \]

\[ \text{Answer: There is about an 8.3\% chance of selecting 3 women for this committee.} \]

\[ \text{With and Without Replacement} \]

As you saw in the previous two examples, whenever objects (or people) are chosen at random, the probability changes for the second pick, if the object (or person) is not put back. In probability, putting an object (or person) back in the bag (or applicant pool, or whatever) is called \textit{replacement}.

\[ \text{Question 26: Suppose you have a bag containing 3 red tiles and 7 white tiles.} \]

\[ a) \text{ Find the theoretical probability of getting two red tiles in a row, if you replace the first tile before drawing the second. Are the events of selecting two red tiles independent if we return the first one to the bag prior to selecting the second button?} \]

\[ b) \text{ Find the theoretical probability of getting two red tiles in a row, if you do not replace the first draw. Are the events of selecting two red tiles independent if we are not returning the first tile prior to selecting the second one?} \]

\[ \text{In summary:} \]

\[ \text{With replacement means the object is put back, and the events are independent} \]

\[ \text{Without replacement means the object is not put back and the events are dependent} \]
Lesson 6: Probability Distributions of Discrete Random Variables

Notation
Suppose you pick a student at random and ask him or her how many times per day he or she checks his or her email. Let the random variable $X$ = "number of times a person checks his or her email"

Explain each of the following:

- $X > 8$
- $X \leq 4$
- $3 < X \leq 5$
- $P(X > 4)$
- $P(2 \leq X \leq 5)$

Probability Distribution with Dice

Let a trial be rolling 2 dice
Let the random variable $X$ = the result of rolling two dice.

For example,
With this dice roll:

X = 10

Question 27: What are the highest and lowest values of $X$?

Question 28: There are 36 outcomes when rolling two dice. Below is a list of all 36 outcomes:

![List of all 36 outcomes when rolling two dice]

Question 29: Based on the table above, calculate $P(X = 2)$

Question 30: What is the most likely value of $X$? Why?
**Question 31:** We will now construct a *probability distribution*. Calculate the probability of each possible value of $X$. The first row is done for you.

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<thead>
<tr>
<th>$X$</th>
<th>Probability (fraction form)</th>
<th>Probability (decimal form)</th>
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<tbody>
<tr>
<td>2</td>
<td>$\frac{1}{36}$</td>
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**Question 32:** Next, make a histogram for the random variable $X$, then describe it. Then, explain what you think is meant by the phrase *probability distribution*.

**Question 33:** Describe the center, shape, and spread of the probability distribution.

**Question 34:** Based on the probability distribution, what is $P(X = 4)$?

**Question 35:** Based on the probability distribution, what is $P(5 < X < 10)$?

**Question 36:** Based on the probability distribution, calculate $P(3 < X < 8)$
Question 37: Next, we’ll test the theory. With your classmates, roll 2 dice over and over and use tally marks (|||) to keep track of your results.

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Question 38: Combine your results with the whole class, and compute the relative frequency of each value of X as well:

<table>
<thead>
<tr>
<th>X</th>
<th>Frequency</th>
<th>relative frequency</th>
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Question 39: Compare the center, shape, and spread of the theoretical probability distribution (from questions 5, 6, and 7) to the empirical probability distribution (from question 13). Are they close?

Question 40: Explain, in the context of probability, what the center of the distribution means.

Question 41: Explain, in the context of probability, what the spread of the distribution means.

Question 42: The theoretical probability and the empirical probability should be similar. This is another example of the **Law of Large Numbers**. Explain in your own words what you think this means.
YOU NEED TO KNOW

A *discrete probability distribution* is a list of all the possible values of the discrete random variable together with each value’s probability. Each probability is between 0 and 1, and the sum of all probabilities is 1.

TRY THESE

Suppose a family is chosen at random, and the number of children a family has is given by the probability distribution below.

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<tr>
<th>Number of children</th>
<th>Probability</th>
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<tr>
<td>0</td>
<td>0.2</td>
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<tr>
<td>1</td>
<td>0.25</td>
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<tr>
<td>2</td>
<td>0.35</td>
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<td>3</td>
<td>0.1</td>
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<tr>
<td>4</td>
<td>0.05</td>
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<tr>
<td>5</td>
<td>0.05</td>
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**Question 43:** Make a histogram of this probability distribution, then describe the center, shape and spread.

**Question 44:** Let $X$ = the number of children. Explain what $P(X > 3)$ means, then compute it.

**Question 45:** Explain what $P(2 < X < 5)$ means, then compute it.

**Question 46:** Compute the area of the bar centered at 3, and also give $P(X=3)$. What is the relationship between area and probability?
Lesson 7: Area as Probability

In some circumstances, area can be used to compute probability. This is especially useful when dealing with probability distributions.

Consider the following random trial: Suppose you throw a dart towards a dartboard (without aiming). The dart will hit a random location somewhere on the grid below:

What is the probability of hitting the dartboard? Here are some hints:

1. Let’s assume each small square on the grid is one inch wide. So the entire grid is 16 inches tall and 16 inches wide
2. The formula for the area of a circle is $A = \pi r^2$

This brings us to a general formula for using area to find probability:

Let event A be the event that you “hit” area A. Then...

**Question 47:** $P(A) =$ ?
Homework

Lesson 1

Directions: For each statement below
   a) State the random trial
   b) State the event, and give a letter to represent the event
   c) Write the statement using probability notation (give the answer using all three forms: decimal, percent, and fraction notation)

Example:
There is a 30% chance that the mayor will get re-elected.

   a) The random trial is the election
   b) The event is “the mayor gets re-elected”. Let’s call this event R (for re-elected)
   c) P(R) = 0.3

Exercises

1. There is a 75% chance of rain tomorrow
2. The Seattle Seahawks have an 80% chance of getting to the playoffs
3. The chances of rolling a two is 1 out of 36 (hint: write the answer as a fraction)
4. If I flip a coin, there is a 50% chance of getting heads.

Directions: Let the random trial be rolling 2 dice. Compute the following probabilities. Use the dice chart from this lesson to count the number of ways the event can occur, then divide by 36. Write your answer using probability notation (such as P(X=4)) and write your answer in decimal, fraction, and percent form.

5. The probability of rolling a 6
6. The probability of rolling a 12
7. P(rolling a 2)
8. P(rolling a 3)
9. P(X = 9)
10. P(X = 4)
11. P(rolling a number more than 10)
12. P(rolling a number less than 5)
Lesson 2

13. A bag contains 5 purple marbles and 7 orange marbles. You pick one marble at random, note the color, then put it back in the bag.

   A) What is the probability of picking a purple marble?

   B) If you repeated this trial 1,200 times, approximately how many purple marbles would you pick?

14. A bag contains 9 purple marbles and 3 orange marbles. You pick one marble at random, note the color, then put it back in the bag.

   A) What is the probability of picking a purple marble?

   B) If you repeated this trial 1,200 times, approximately how many purple marbles would you pick?

15. A baseball player has been “at bat” (i.e. attempted to hit the ball) 369 times. He has hit the ball 165 times. What is this player’s probability of hitting the ball? Is this an empirical or theoretical probability?

16. A football player has caught the ball 67 times out of 192 attempts. What is this player’s probability of catching the ball? Is this an empirical or theoretical probability?

17. If you roll one die, the probability of rolling a five is 1/6. If you rolled the die 6000 times, approximately how many times would you predict rolling a five?

18. If you roll one die, the probability of rolling a two is 1/6. If you rolled the die 3000 times, approximately how many times would you predict rolling a two?

19. A car insurance company knows the probability of a person getting in a car accident is 5%. If the insurance company insures 4,000,000 customers, approximately how many of them will be involved in a car accident?

20. A fire insurance company knows that there is a 0.2% chance of a fire. If the company insures 2,000,000 homes, approximately how many fires will there be?

Lesson 3

21. A coin is flipped three times. What is the probability that exactly two tails come up? Circle the appropriate outcomes below, and calculate the answer:

   HHH   HHT   HTH   HTT   THH   THT   TTH   TTT

22. A coin is flipped three times. What is the probability that exactly one head comes up? Circle the appropriate outcomes below, and calculate the answer:

   HHH   HHT   HTH   HTT   THH   THT   TTH   TTT
23. A coin is flipped 2 times. Write out the sample space, then calculate the probability of getting 2 tails.

24. Suppose you are rolling two 7 sided dice, each with the numbers 1-7 for sides.
   A) How many possible outcomes are there?
   B) List all the possible outcomes: \((Hint: \text{it may help to think as Die #1 as red, Die #2 as blue})\)
   C) List the theoretical probability of rolling a...
      
      \begin{align*}
      2 & \quad 8 & \quad 12 & \quad 14 \\
      \text{number less than 5} & \quad \text{number greater than 5, but less than 10} & \quad \text{NOT rolling a 5} \\
      \end{align*}

25. The PIN (personal identification number) of a debit card has 4 digits. If a thief has one chance to guess the correct PIN, what is the probability of a correct guess?

26. Some debit cards have a 6-digit PIN. If a thief has one chance to guess the correct PIN, what is the probability of a correct guess?

Lesson 4

27. A bowler has a 25% chance of getting a strike (knocking all the pins down in one roll of the ball), and a 35% chance of getting a spare (knocking all the pins down in two rolls).
   a) What is the probability of not getting a spare?
   b) What is the probability of getting a strike or a spare?

28. A baseball player has a 30% chance of hitting the ball, and a 10% chance of getting a strike-out.
   a) What is the probability of getting a hit or a strike-out?
   b) What is the probability of not striking out?

29. A special 4-sided die, labeled A, B, C, and D, has the following probabilities: \(P(A) = 0.3\), \(P(B) = 0.4\), \(P(C) = 0.1\), \(P(D) = 0.2\). Is this a valid set of probabilities, based on the rules of probability? Why or why not?
30. A special 4-sided die, labeled A, B, C, and D, has the following probabilities: \( P(A) = 0.3, P(B) = 0.3, P(C) = 0.3, P(D) = 0.3 \). Is this a valid set of probabilities, based on the rules of probability? Why or why not?

31. When playing \textit{Toss the Pigs}, what is the probability that a pig lands on a side (either the left or right side)?

32. When playing \textit{Toss the Pigs}, what is the probability that a pig does not land on a side (neither the left nor right side)?

33. When rolling two dice, what is the probability of not getting a 12?

34. When rolling two dice, what is the probability of not getting a 7?

35. A bag contains 5 red marbles, 4 green marbles, 2 yellow marbles and 6 blue marbles. Use the rules of probability to compute the following:
   a) The probability of not getting a green marble
   b) The probability of getting a green or red marble
   c) Show that the sum of all the probabilities is 1. (in other words, show that \( P(\text{red}) + P(\text{green}) + P(\text{yellow}) + P(\text{blue}) = 1 \))

36. A bag contains 3 red marbles, 6 green marbles, 5 yellow marbles and 1 blue marble. Use the rules of probability to compute the following:
   a) The probability of not getting a green marble
   b) The probability of getting a green or red marble
   c) Show that the sum of all the probabilities is 1. (in other words, show that \( P(\text{red}) + P(\text{green}) + P(\text{yellow}) + P(\text{blue}) = 1 \))

\textbf{Lesson 5}

37. A quiz has 3 true/false questions. If a student doesn’t know the answers and just randomly guesses, what is the probability that all 3 questions are answered correctly?

38. A quick quiz consists of a multiple-choice question with 5 possible answers followed by a multiple-choice question with 6 possible answers. If both questions are answered with random guesses, find the probability that both responses are correct.

39. Suppose that 13% of people own dogs. If you pick two people at random, what is the probability that they both own a dog?

40. Suppose 25% of people prefer Pepsi over Coke. If two people are chosen at random, what is the probability that both prefer Pepsi over Coke?
41. A bag contains 4 red tiles, 5 blue tiles, and 3 white tiles. Compute the probability of the following events

   a) Drawing a red tile
   b) Drawing two red tiles in a row (without replacement)
   c) Drawing two red tiles in a row (with replacement)
   d) Drawing two tiles and neither one being red (hint: \textit{not getting red} twice in a row); with replacement
   e) Drawing two tiles and neither one being red (hint: \textit{not getting red} twice in a row); without replacement

42. A bag contains 7 red tiles, 6 blue tiles, and 2 white tiles. Compute the probability of the following events:

   a) Drawing a red tile
   b) Drawing two red tiles in a row (without replacement)
   c) Drawing two red tiles in a row (with replacement)
   d) Drawing two tiles and neither one being red (hint: \textit{not getting red} twice in a row); with replacement
   e) Drawing two tiles and neither one being red (hint: \textit{not getting red} twice in a row); without replacement

\textbf{Lesson 6}

\textit{LET THE TRIAL BE FLIPPING 3 COINS. BELOW IS THE SAMPLE SPACE:}

\begin{center}
HHH HHT HTH HTT THH THT TTH TTT
\end{center}

Let the random variable \(X = \) the number of tails (out of 3 coins). Complete the probability distribution below:

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
\(X\) & Probability (fraction form) & Probability (decimal form) \\
\hline
0 & & \\
1 & & \\
2 & & \\
3 & & \\
\hline
\end{tabular}
\end{center}

43. Make a histogram of the probability distribution from the previous question.
44. Suppose there is a pair of special dice, each with 4 sides. Let $X =$ the sum of the two dice. (the lowest is double-ones, where $X = 2$, the highest is double-fours, where $X = 8$). Write the sample space, make a table showing the probability distribution, then make a histogram.

45. Suppose a person is selected at random and is asked how many movies he or she goes to per month. Let the random variable $X =$ the number of movies a person watches goes to in one month. Below is a probability distribution.

<table>
<thead>
<tr>
<th>Number of Movies</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.10</td>
<td>0.10</td>
<td>0.24</td>
<td>0.30</td>
<td>0.14</td>
<td>0.12</td>
</tr>
</tbody>
</table>

a) Explain what $P(X=2)$ means, then give the probability

b) Explain what $P(X < 3)$ means, then compute it

c) Make a histogram of this probability distribution, then describe the center, shape, and spread.

d) What is the area of the bar centered at 2? Explain why this is the same as the answer to question 1.

Lesson 7

46. A dart will hit somewhere on the grid below. Find the probability of hitting the stop-sign.