I. Applying Newton’s laws to liquids

A rectangular container filled with water is at rest on a table as shown. Two imaginary boundaries that divide the water into three layers of equal volume have been drawn in the diagram. (No material barrier separates the layers.)

![Free-body diagrams]

A. For each layer, draw a free-body diagram in the space provided. Be sure to indicate on your diagram the surface on which each contact force is applied. (This is usually done by placing the tip of the arrow that represents the force at that surface.)

The label for each force should indicate:
- the type of force,
- the object on which the force is exerted, and
- the object exerting the force.

B. Rank the magnitudes of all the vertical forces you have drawn in the three diagrams from largest to smallest. Explain how you determined your ranking.

For any layer (e.g., 2):

\[ N_{21} + W_{2E} = N_{23} \Rightarrow N_{1a} < N_{12} = N_{21} < N_{23} = N_{32} < N_{3B} \]

How does the weight of layer 1 compare to that of layer 3? Also,

But can’t compare \( W_{1E} \) and \( W_{2E} \).

A liquid in which equal volumes have equal weight regardless of depth (i.e., the density does not vary) is referred to as incompressible. Assume that all liquids in this tutorial are incompressible.

C. Imagine that a small hole is opened in the container wall near the bottom of each layer.

1. Predict what will happen to the water near each hole. Explain.

   *It will exit the container through the hole in the form of a jet.*

2. Check your prediction by observing the demonstration. Record your observations. (A sketch may be helpful.)

   What do your observations suggest about: (1) the existence of horizontal forces on the three layers of water in part A? (2) the relative magnitudes of the horizontal forces on the three layers?

   \[ N_{1C L} = N_{1C R} < N_{2C L} = N_{2C R} < N_{3C L} = N_{3C R} \]

   If necessary, revise your free-body diagrams in part A so that they are consistent with your answers.

_Tutorials in Introductory Physics_
II. Pressure and force

A. Recall the relationship between force and pressure. (Consult your textbook if necessary.) Below we will apply this relationship to the three layers from part I.

1. Which force would you use to determine the pressure at the bottom of layer 2? (There may be more than one correct answer.) Explain your reasoning. (Hint: Refer to your free-body diagrams from section I. Which forces are exerted at the bottom of layer 2?)

\[ \vec{N}_{23} \quad (could \ also \ use \ \vec{N}_{32} \quad \text{but} \quad \vec{N}_{32} = \vec{N}_{23}) \]

2. Which area would you use to determine the pressure at the bottom of layer 2? Explain.

The area of the bottom of layer 2

3. Suppose that you wanted to determine the pressure at a point in the center of layer 2. For what object(s) would you draw a free-body diagram? Which force and which area would be useful in determining the pressure?

Create a layer whose bottom (or top) goes through the center. Take the area of the bottom of the layer.

B. Suppose you wanted to determine the pressure at the top surface of layer 1. Which force would you use to determine this pressure? If necessary, modify your free-body diagrams to include this force. Be sure to label your diagram to indicate the object that exerts this force.

\[ \vec{N}_{1a} \]

Three points, L, M, and N, are marked at the bottom of the three layers.

C. Rank the pressures at points L, M, and N. Explain how your answer is consistent with your ranking of forces in section I.

\[ P_N > P_M > P_L \quad (since \quad P = \frac{N}{A}) \]

The pressure \( P \) at a point in an incompressible liquid is often described mathematically as \( P = P_o + \rho gh \).

D. Is your ranking in part C consistent with this equation? (Hint: At what point is \( h = 0 \)? What is the pressure at that point?)

\[ h = 0 \text{ at the surface } \Rightarrow P = P_o \]
III. Pressure as a function of depth

The container at right is filled with water and is at rest on a table. An imaginary boundary that outlines a small volume of water has been drawn in the diagram. Treat this small volume of water as a single object.

A. Draw a free-body diagram for the small volume of water in the space below the figure.

B. Compare the magnitudes of the horizontal forces that you have drawn.

\[ N_{VR} = N_{VL} \]

Is your answer consistent with the motion of the small volume of water? Explain.

Since \( a = 0 \), \( \vec{F}_{\text{net}} = 0 \)

and \( F_{\text{net}, x} = 0 \)

C. Use your answer to part B to compare the pressures at points T and U. (Hint: How is the pressure at point T related to the force on the small volume of water by the water to its left?)

\[ P_T = P_U \]

D. Rank the pressures at points Q, R, S, T, and U. Explain.

2 points at the same depth have the same pressure. The deeper the point is, the higher its pressure is \( \Rightarrow P_Q < P_R = P_S < P_T = P_U \)

E. Consider the following student dialogue:

Student 1: "The pressure at a point is equal to the weight of the water above divided by the area. Therefore the pressure at point R is greater than the pressure at point S because there's no water above point S."

Student 2: "I agree. The pressure is \( P_o + \rho gh \), and \( h \) is zero for point S and greater than zero for point R. Therefore, the pressure at R must be greater."

Do you agree with either student? Explain your reasoning.

I agree with neither: the pressure is the same everywhere on a horizontal plane. \( \Rightarrow P_R = P_S \). Need to take \( h = \Delta z \) between the point and the surface to apply \( P = P_o + \rho gh \).

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IV. Pressure in a U-tube

A U-shaped tube is filled with water as shown.

A. Rank the pressures at points A through F. Explain.

\[ P_F = P_0 < P_A = P_E < P_B < P_C = P_D \]

Is your ranking consistent with the equation \( P = P_0 + \rho gh \)? Explain.

B. The right end of the tube is now sealed with a stopper. The water levels on both sides remain the same. There is no air between the stopper and the water surface.

1. Does the pressure at points A and D increase, decrease, or remain the same? Explain.

Remains the same since the height of water in the open side doesn't change.

Is the pressure at point E greater than, less than, or equal to the pressure at point D?

\[ P_E < P_D \quad (D \text{ is deeper}) \]

Does the difference in pressure \( \Delta P_{DE} \) between points D and E change when the stopper is added? Explain.

\[ \Delta P_{DE} = \rho gh_{DE} \quad \text{no change} \quad \text{so} \quad \Delta P_{DE} \text{remains the same} \]

3. Is the pressure at point F greater than, less than, or equal to atmospheric pressure?

\[ P_F = P_0 \quad \text{same height as the surface on the other side} \]

Is the force exerted by the rubber stopper on the water surface on the right greater than, less than, or equal to the force exerted by the atmosphere on the water surface on the left?

Same since \( P_F = P_0 \)

C. A syringe is used to remove some water from the left side of the U-tube. The water level on the left side is seen to be lowered, but the water level on the right does not change.

Consider the following student dialogue:

Student 1: “The pressure at point F must now be higher than atmospheric pressure because the water there is being pushed up against the stopper.”

Student 2: “I think that the pressure at point E must be the same as at point A because they are at the same level. These points are both at atmospheric pressure. So the pressure at point F is lower than atmospheric pressure because we know that pressure gets less as you go up.”

Student 3: “But water is more dense than air so the pressure at F cannot be less than atmospheric pressure.”

With which student(s), if any, do you agree?

\[ \text{Student 2 is right} \quad (P_F = P_E - \rho gh_{FE}) \]